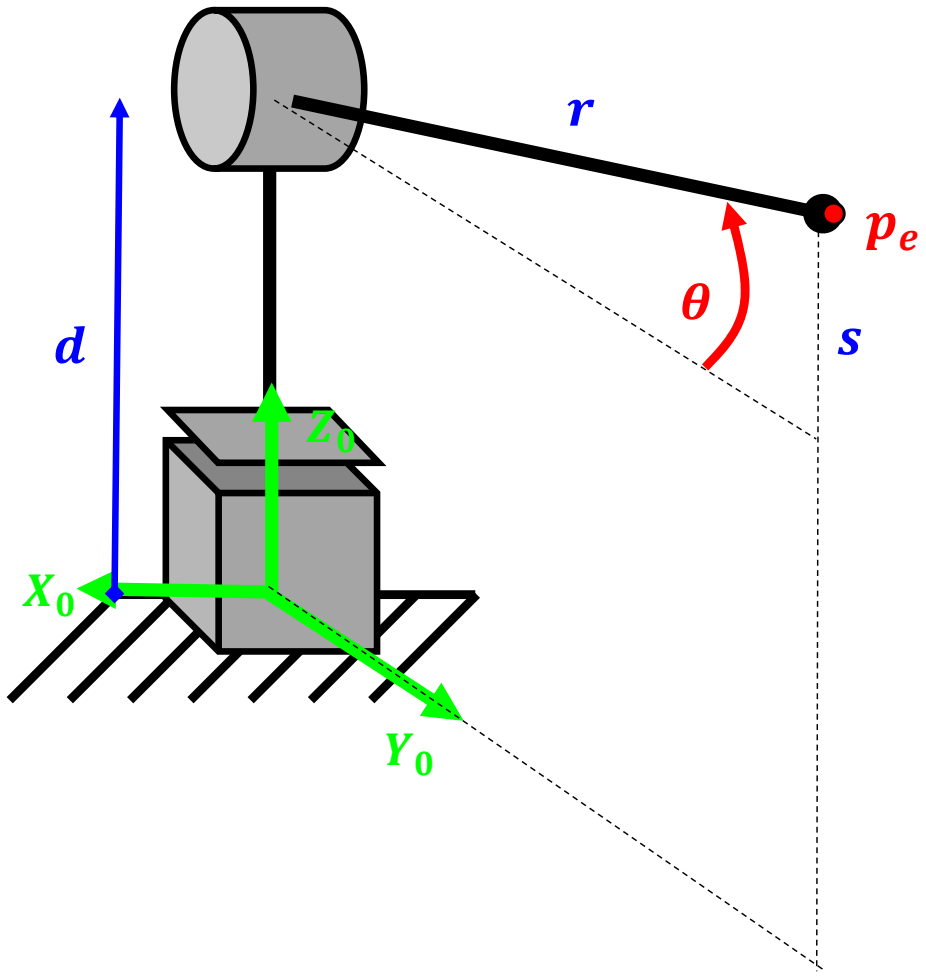
Example – PR Manipulator 1

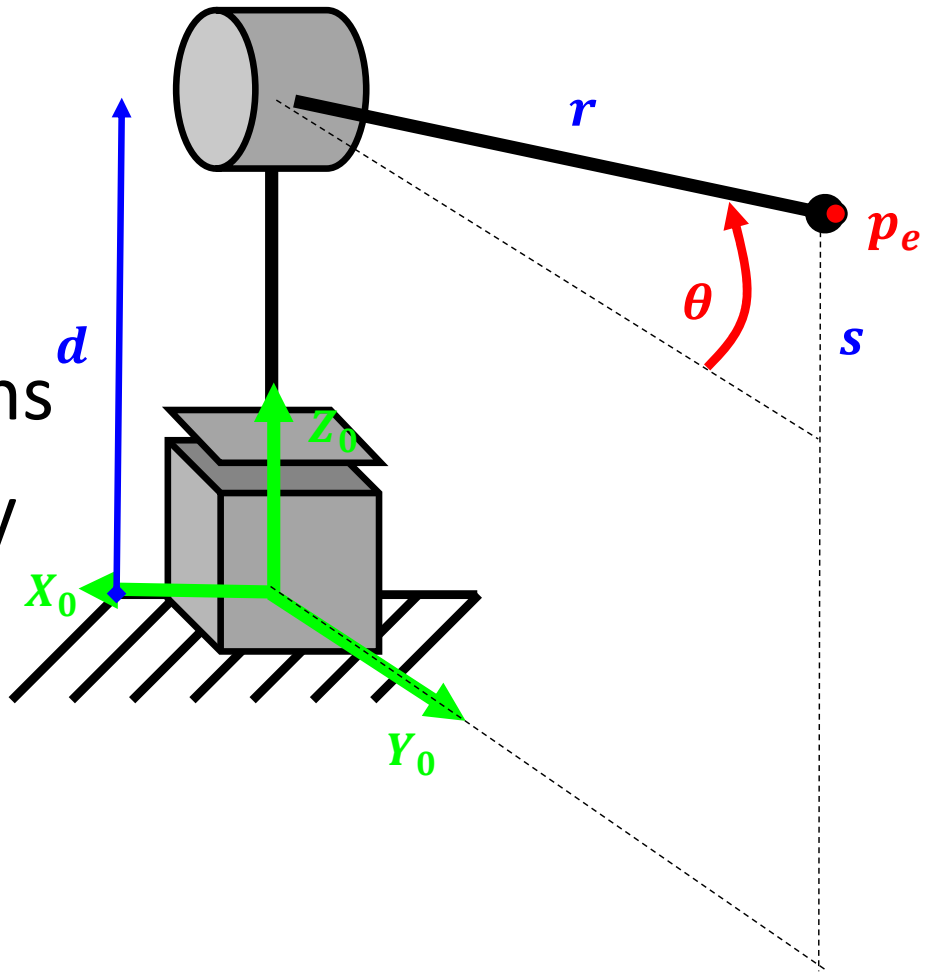
- Given: r, p_e
- Find: singularities (d, θ values)



Example – PR Manipulator 1

- Given: r, p_e
- Find: singularities

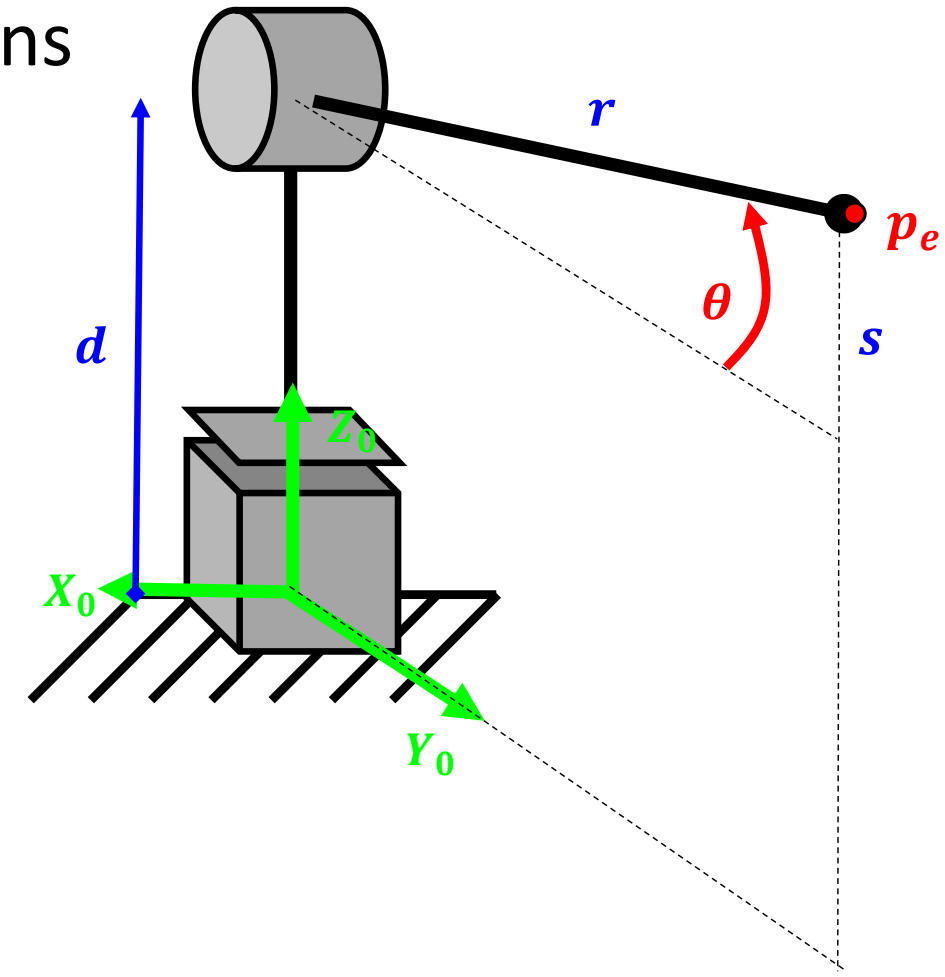
1) Write position FK equations
2) Take derivative for velocity
3) Put in matrix form $\dot{X} = J\dot{\theta}$
4) Set det(J) = 0, solve



Example – PR Manipulator 1

1) Write position FK equations

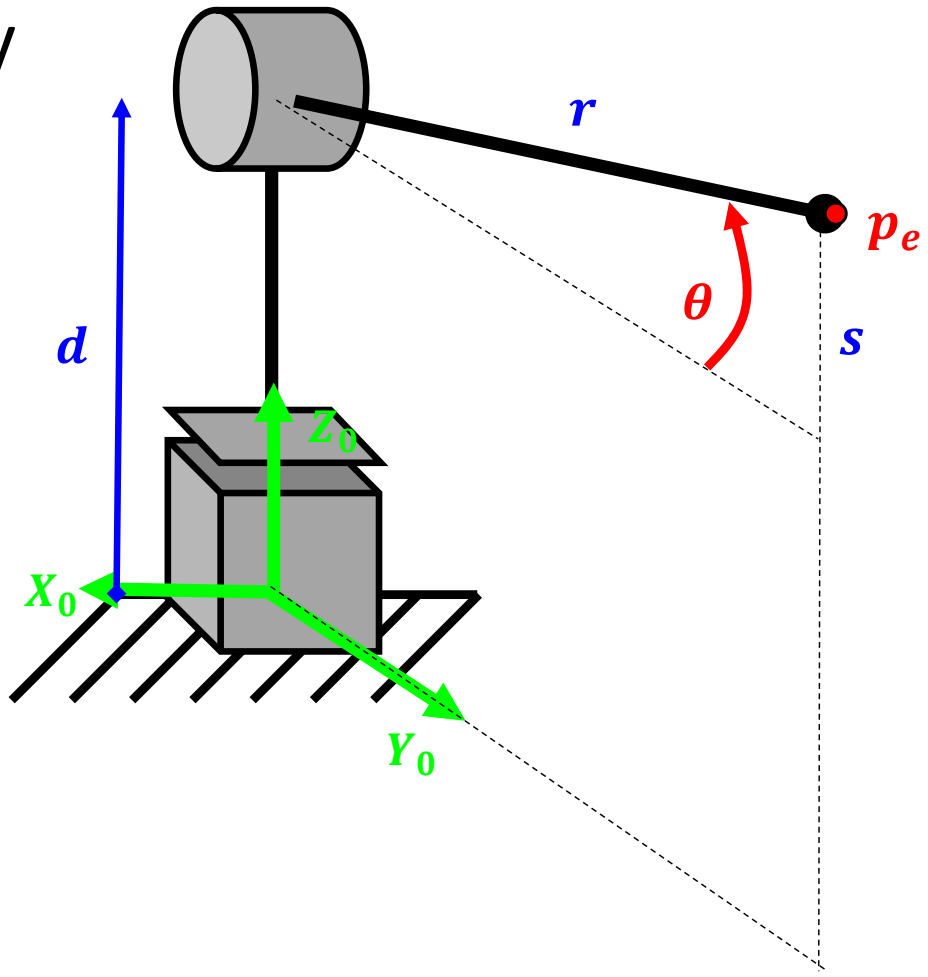
$$x = 0$$
$$y = r\cos(\theta)$$
$$z = d + r\sin(\theta)$$



Example – PR Manipulator 1

2) Take derivative for velocity

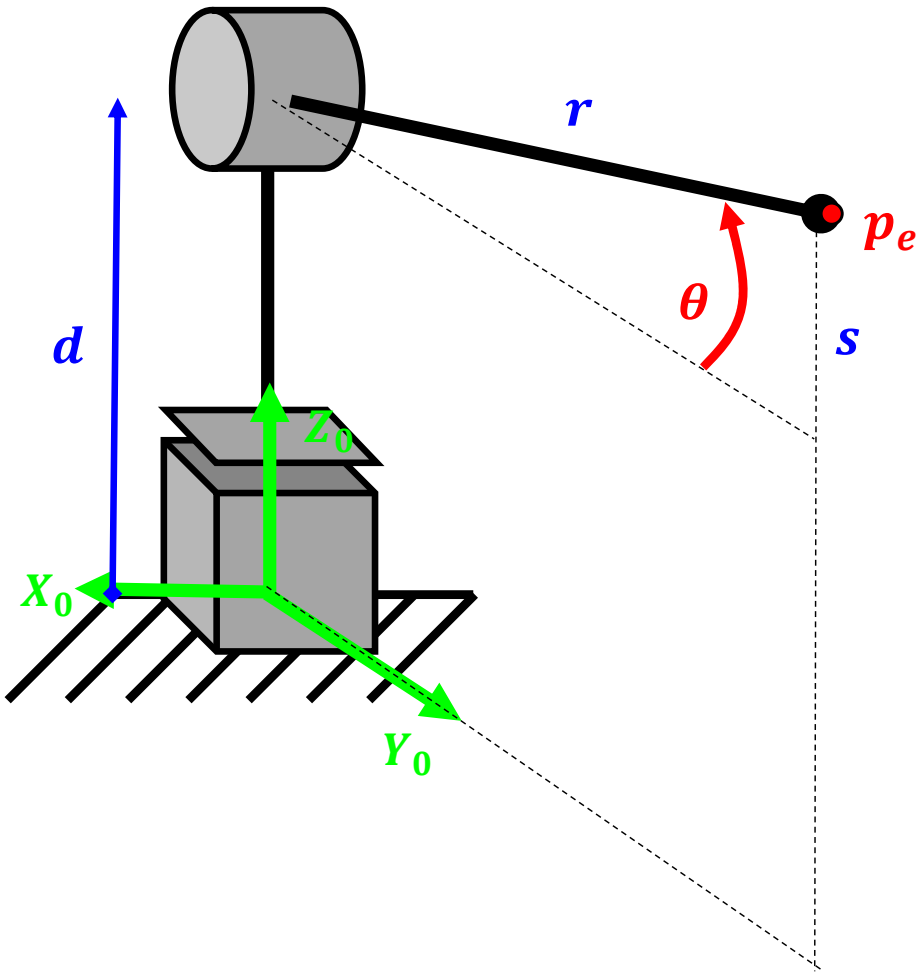
$$\dot{y} = -r\dot{\theta}sin(\theta)$$
$$\dot{z} = \dot{d} + r\dot{\theta}\cos(\theta)$$



Example – PR Manipulator 1

3) Put in matrix form $\dot{X} = J\dot{\theta}$

$$\begin{bmatrix} \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & -r\sin(\theta) \\ 1 & r\cos(\theta) \end{bmatrix} \begin{bmatrix} \dot{d} \\ \dot{\theta} \end{bmatrix}$$

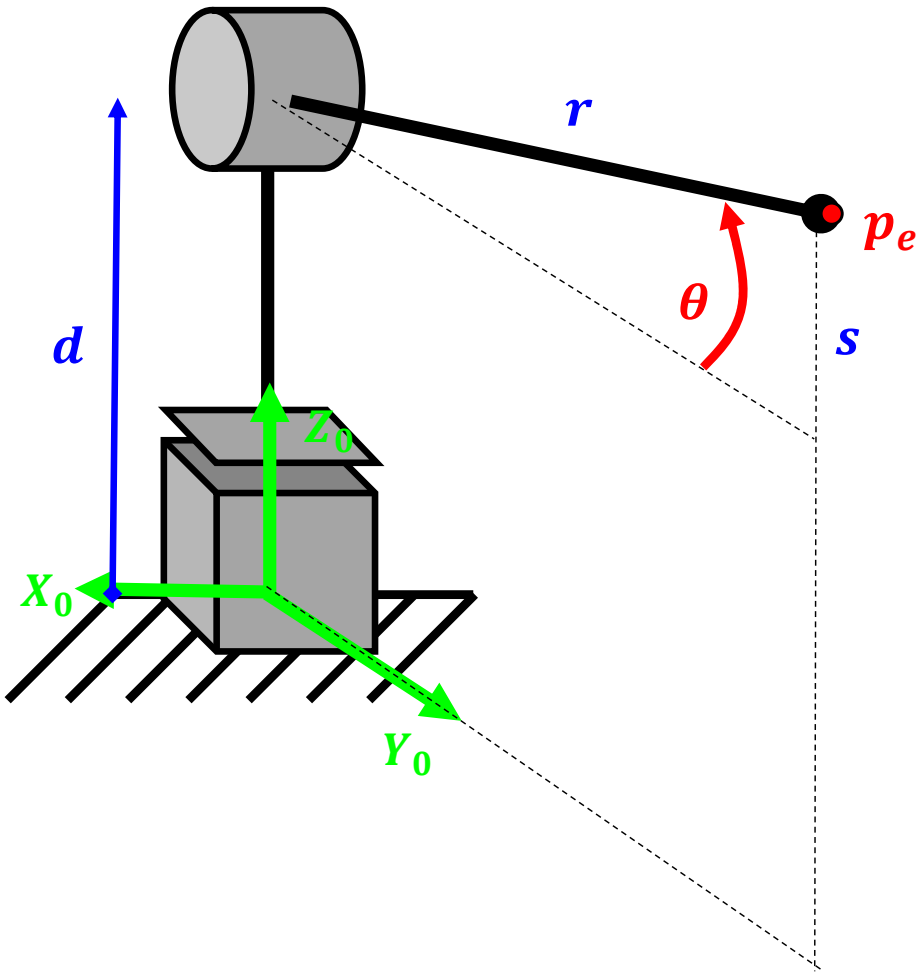


Example – PR Manipulator 1

4) Set det(J) = 0, solve

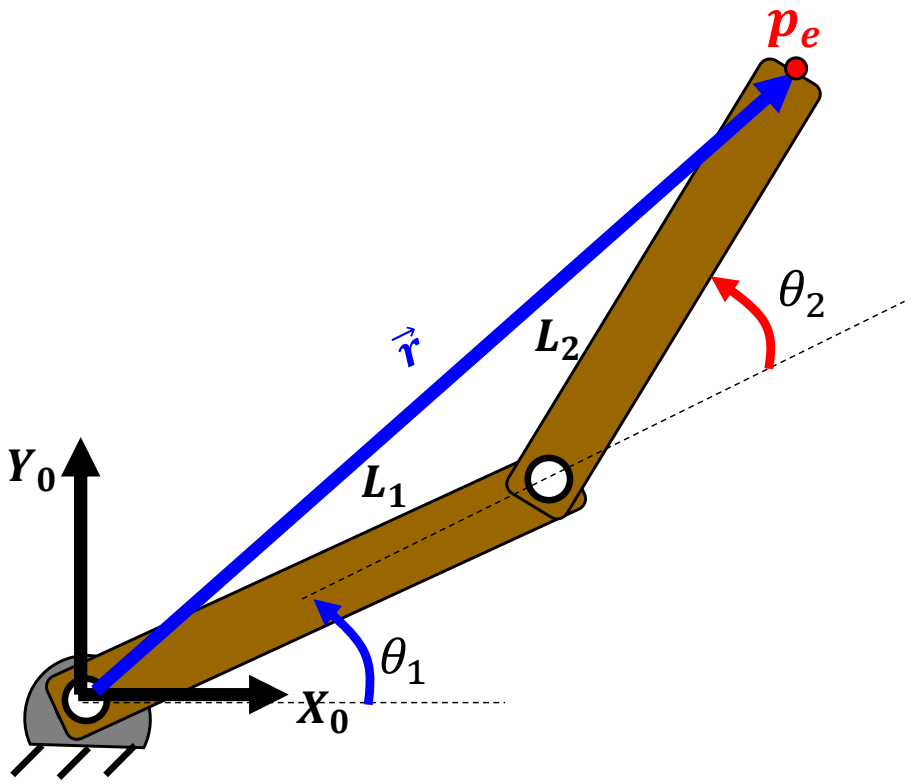
$$rsin(\theta) = 0$$

$$\theta = 0, \pi$$



Example – 2-Link Planar Arm

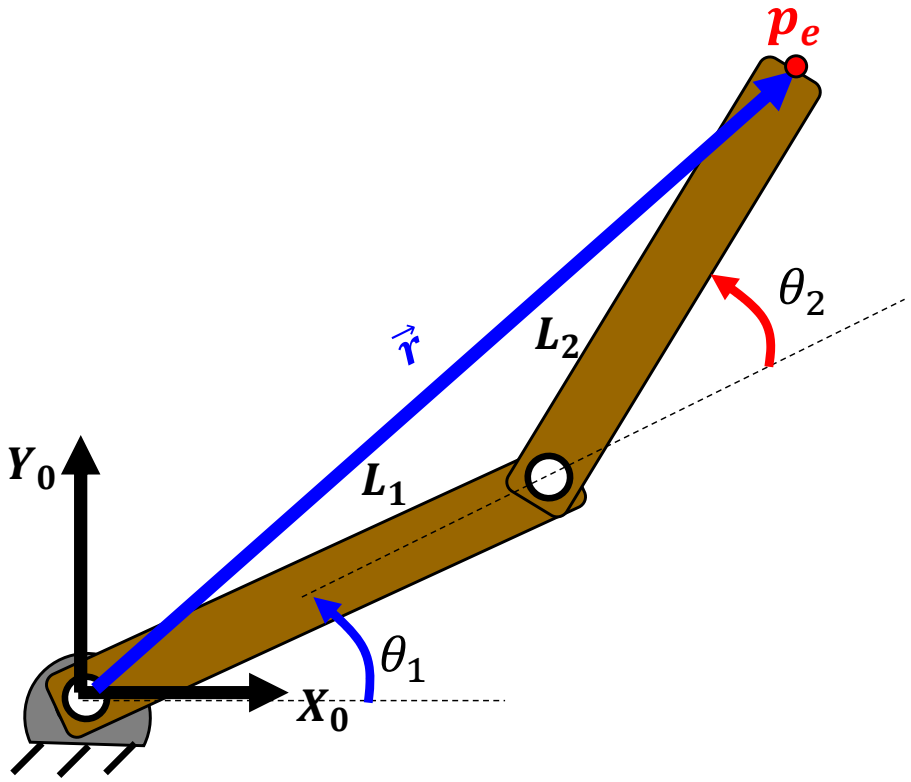
- Given $\mathrm{L}_1,\ L_2,\ and\ p_e = (x_e, y_e)$
- Find: Singularities



Example – 2-Link Planar Arm

- Given $\mathrm{L}_1,\ L_2,\ and\ p_e = (x_e, y_e)$
- Find: Singularities

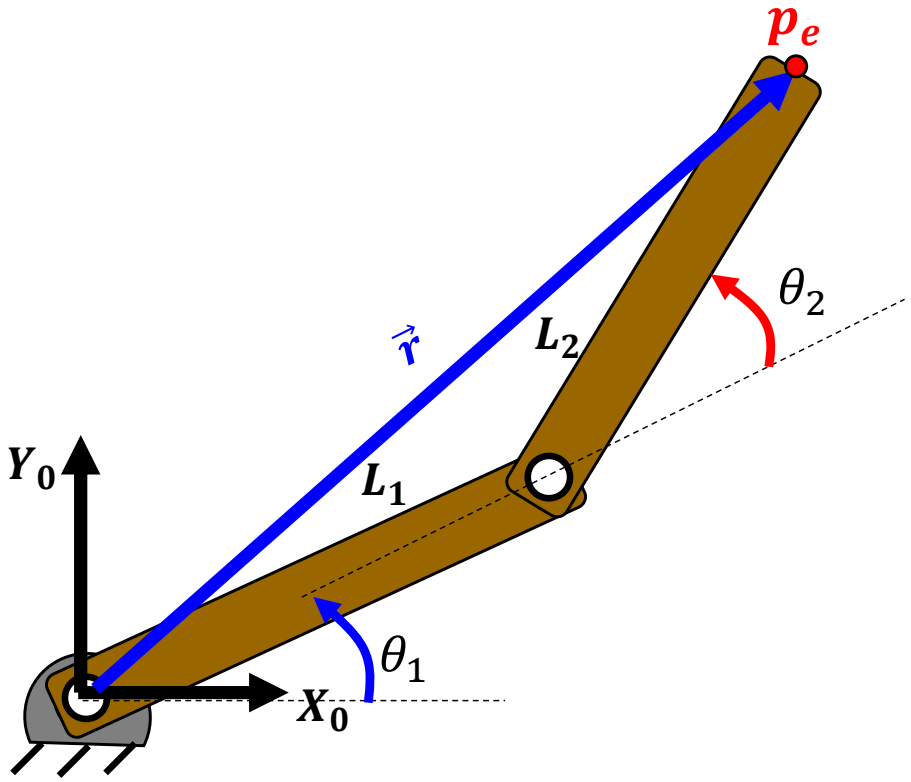
- Inspection: $\theta_2 = 0, \pi$



Example – 2-Link Planar Arm

1) Write position FK equations

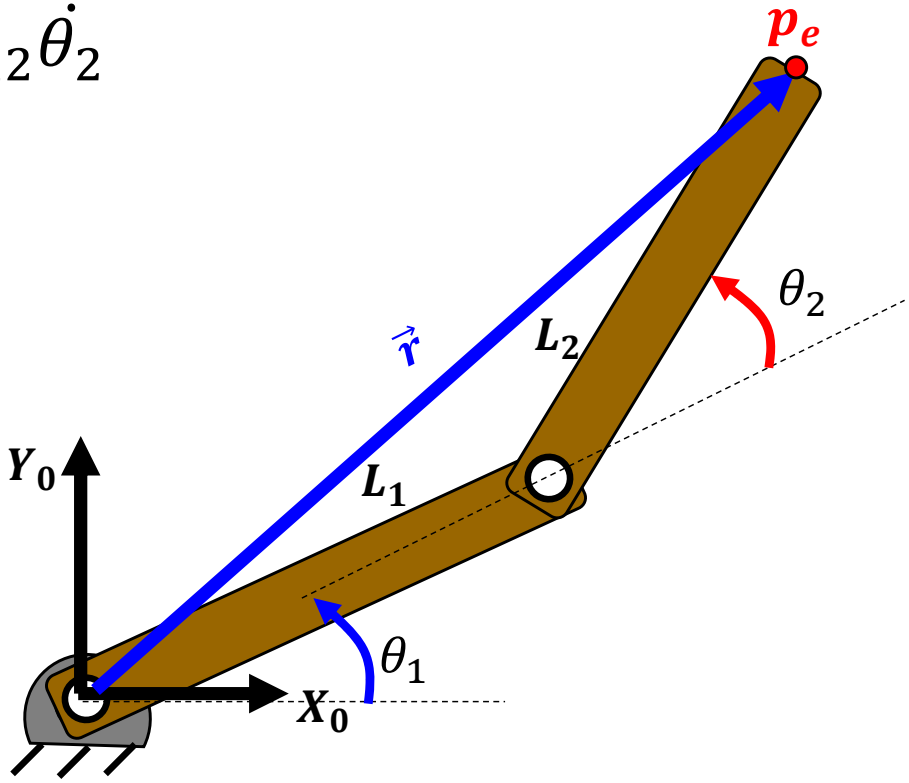
$$x = L_1 c_1 + L_2 c_{12}$$
$$y = L_1 s_1 + L_2 s_{12}$$



Example – 2-Link Planar Arm

2) Take derivative for velocity

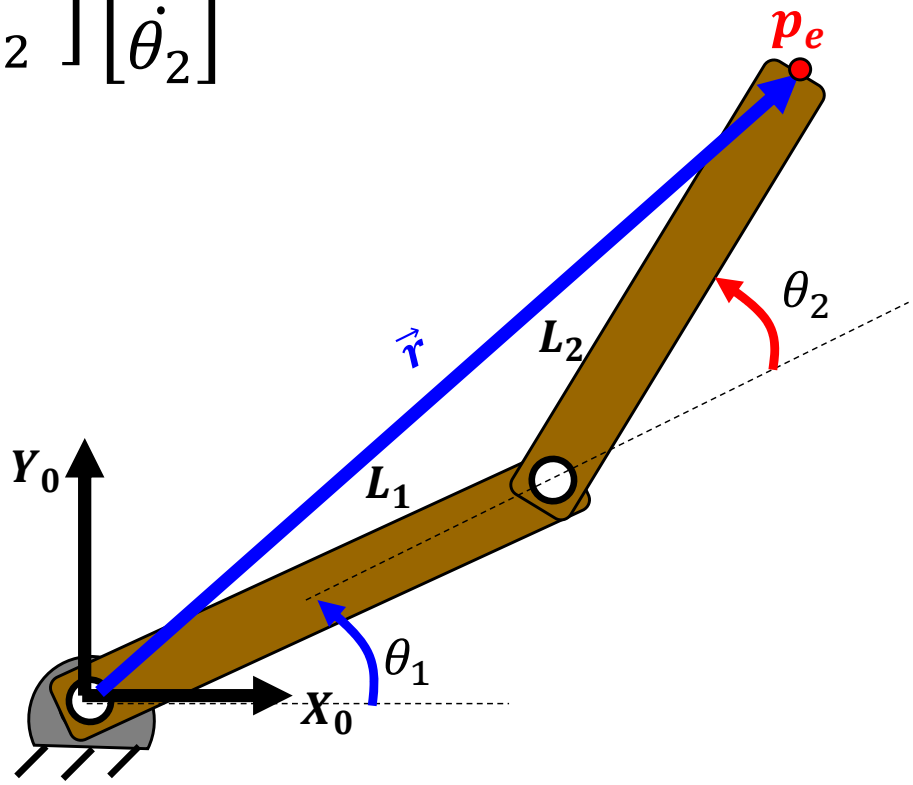
$$\dot{x} = (-L_1 s_1 - L_2 s_{12})\dot{\theta}_1 - L_2 s_{12}\dot{\theta}_2$$
$$\dot{y} = (L_1 c_1 + L_2 c_{12})\dot{\theta}_1 + L_2 c_{12}\dot{\theta}_2$$



Example – 2-Link Planar Arm

3) Put in matrix form $\dot{X} = J\dot{\theta}$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}-L_1 s_1 - L_2 s_{12} & -L_2 s_{12}\\ L_1 c_1 + L_2 c_{12} & L_2 c_{12}\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$$



Example – 2-Link Planar Arm

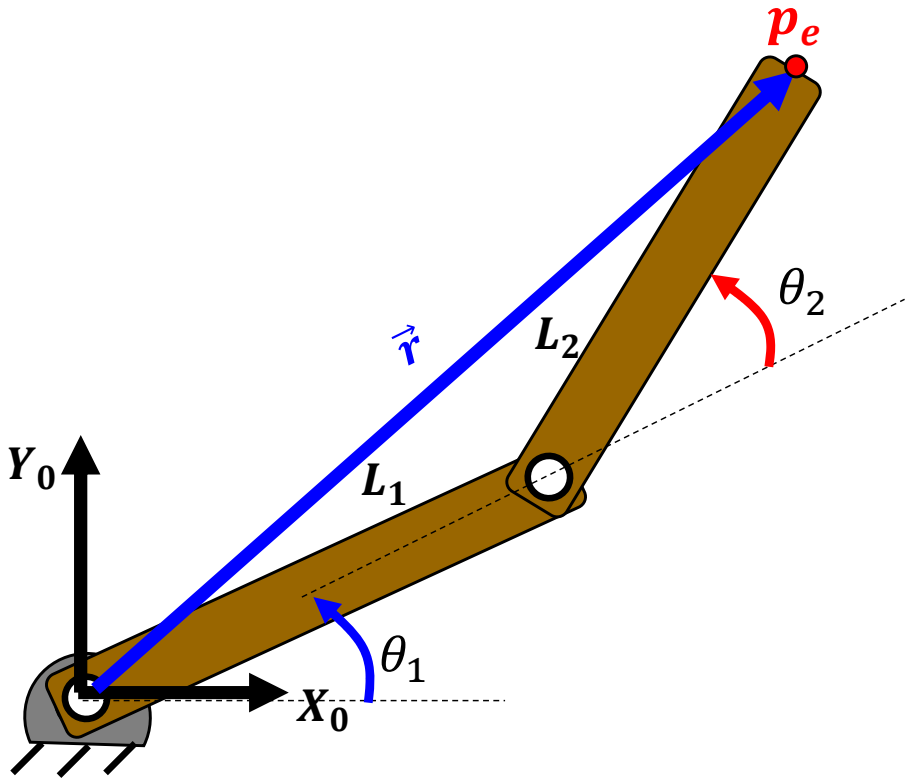
4) Set det(J) = 0, solve

$$0 = (-L_1 s_1 - L_2 s_{12})(L_2 c_{12}) - (L_1 c_1 + L_2 c_{12})(-L_2 s_{12})$$

$$\theta_2 = 0, \pi$$

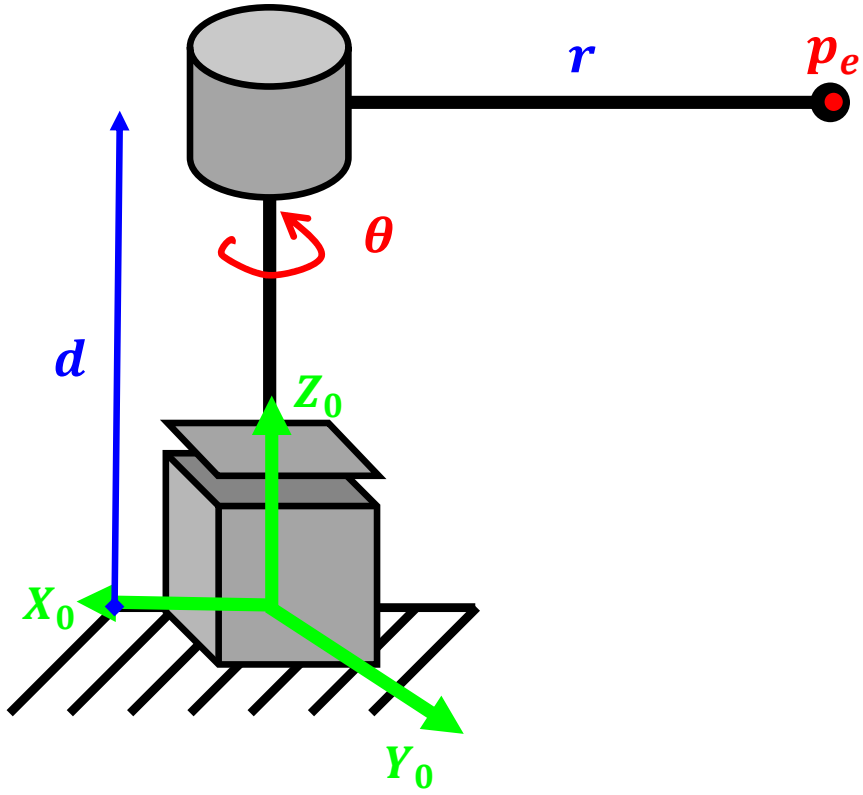
Use trig identity

$$\sin(A + B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$



Example – PR Manipulator 2

- Given: r, p_e
- Find: singularities
  (d, θ values)



Example – PR Manipulator 2

- By inspection:
  no singularities
- By math:
  J = 0 no matter what
  (2 DoF < 3D space)