# Formula Method

- For an n-link robot with joint variables $q_1 \ldots q_n$, With forward kinematics:

$$T_n^0 = \begin{bmatrix} R_n^0 & o_n^0 \\ 0 & 1 \end{bmatrix}$$

0) Determine FK (if not given)
1) Locate all joint origins $o_i^0$
2) Write all joint axis vectors $z_i$
3) Do math by category ($J_v$, $J_\omega$, R joint, P joint)

# Formula Method

- Split into linear $J_v$ and angular $J_\omega$ $\quad J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$
- For $J_v$
  - Assume all joints but *i* fixed
  - Look at end effector velocity

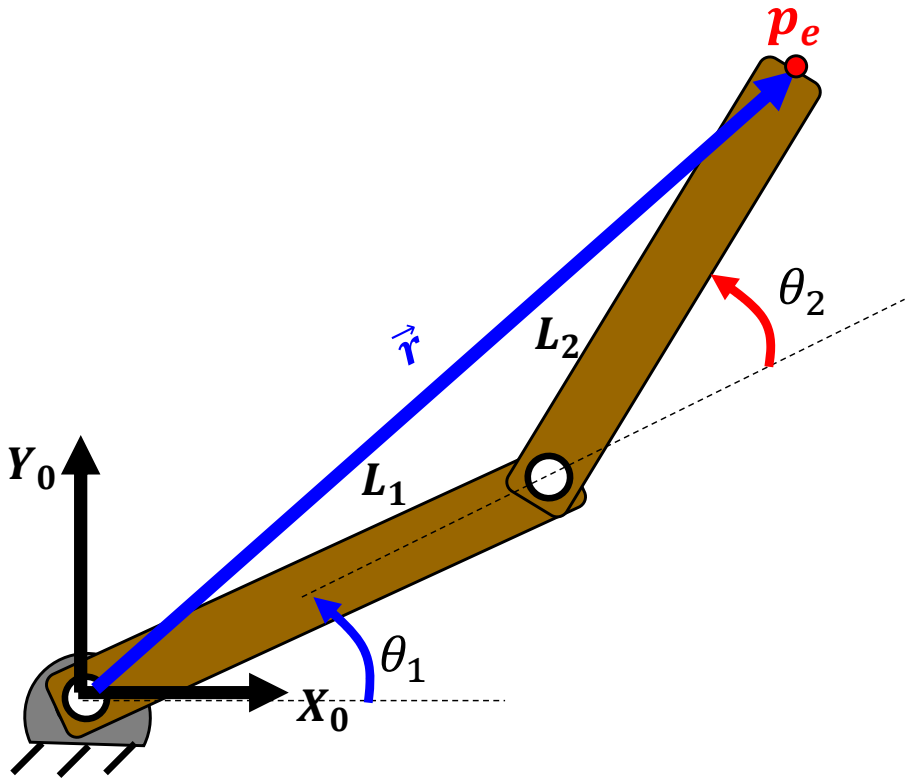
$$J_v = \begin{cases} z_{i-1} \times (o_n - o_{i-1}) & : revolute \\ z_{i-1} & : prismatic \end{cases}$$

- For $J_\omega$
  - Look at frame *i* rotation relative to frame *i-1*

$$J_\omega = \begin{cases} z_{i-1} & : revolute \\ 0 & : prismatic \end{cases}$$

# Example – 2-Link Planar Arm

- Given $L_1,\ L_2$
- Find Jacobian using formula method

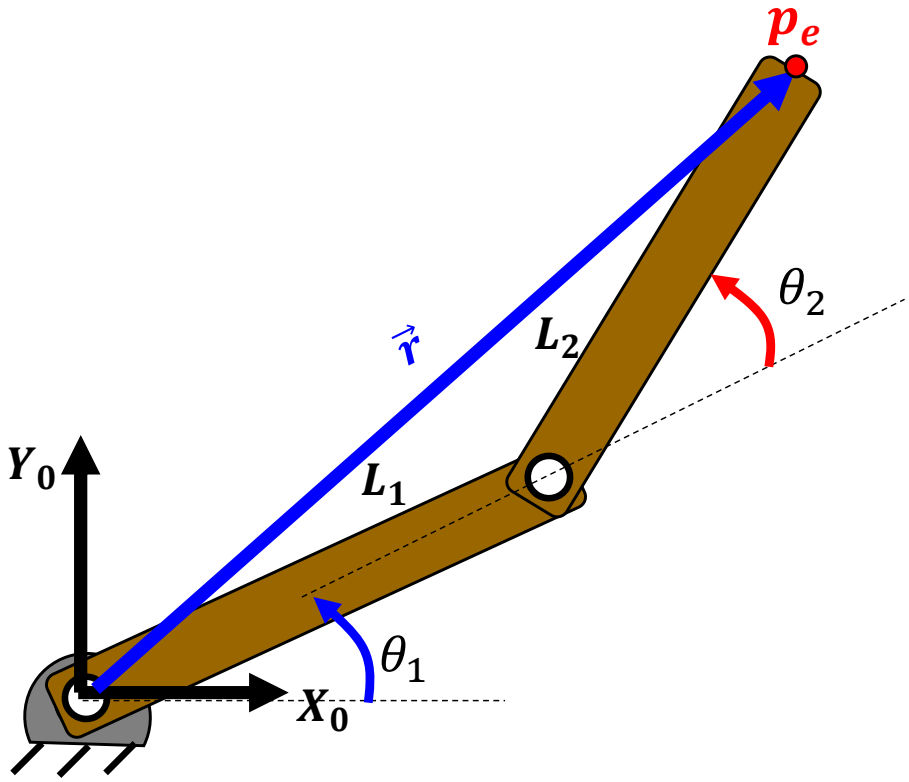


# Example – 2-Link Planar Arm

1) Locate all joint origins $o_i^0$

$$o_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad o_1^0 = \begin{bmatrix} L_1 c_1 \\ L_1 s_1 \\ 0 \end{bmatrix}$$

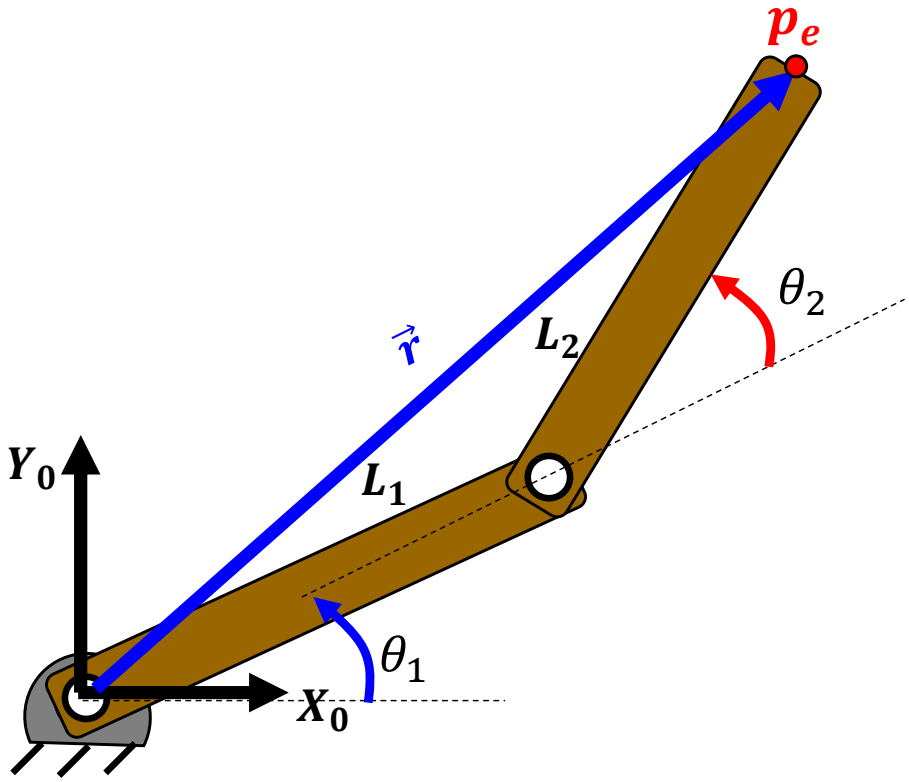
$$o_2^0 = \begin{bmatrix} L_1 c_1 + L_2 c_{12} \\ L_1 s_1 + L_2 s_{12} \\ 0 \end{bmatrix}$$



# Example – 2-Link Planar Arm

2) Write all joint axis vectors $z_i$

$$z_0^0 = z_1^0 = z_2^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

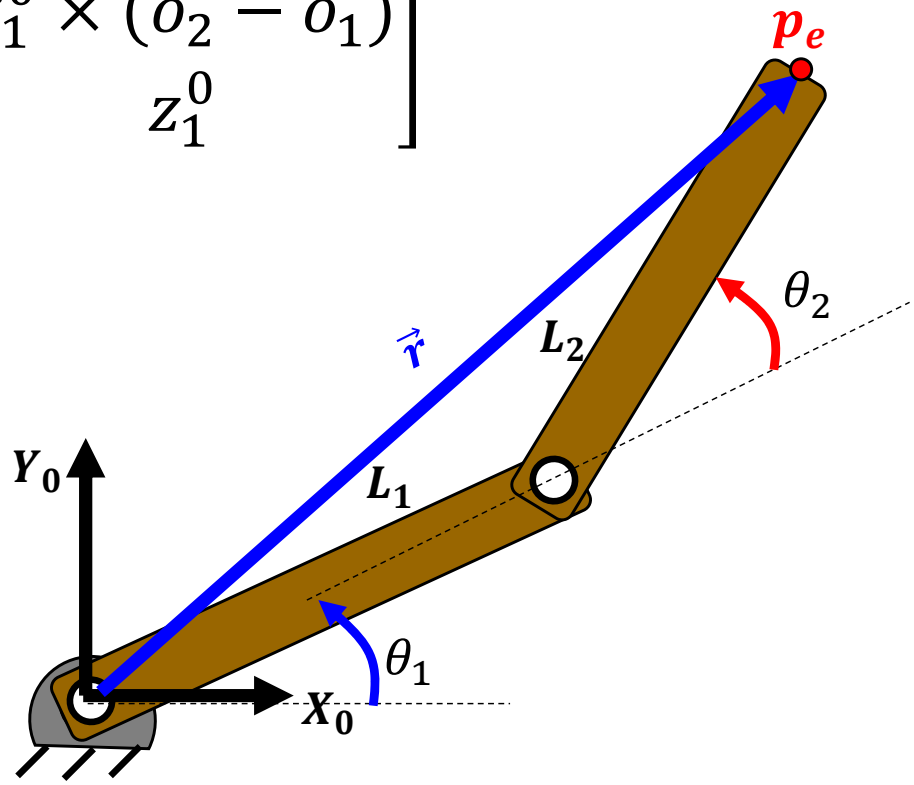


# Example – 2-Link Planar Arm

3) Do math by category (2 R joints)

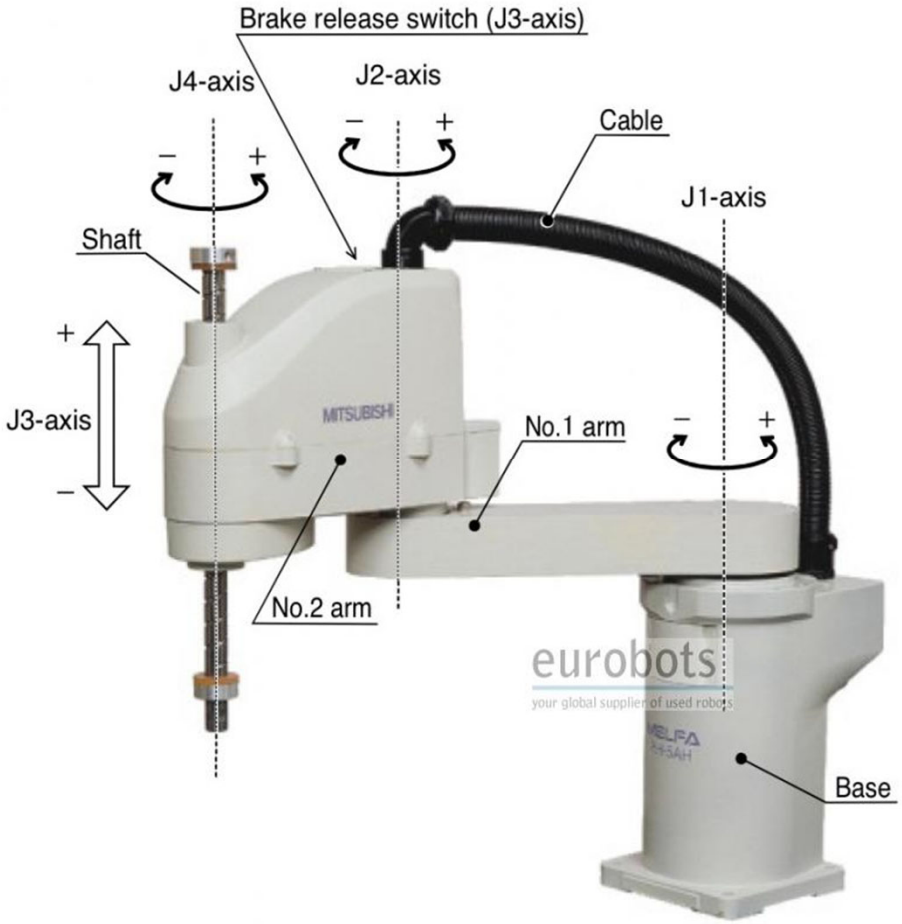
$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} z_0^0 \times (o_2 - o_0) & z_1^0 \times (o_2 - o_1) \\ z_0^0 & z_1^0 \end{bmatrix}$$

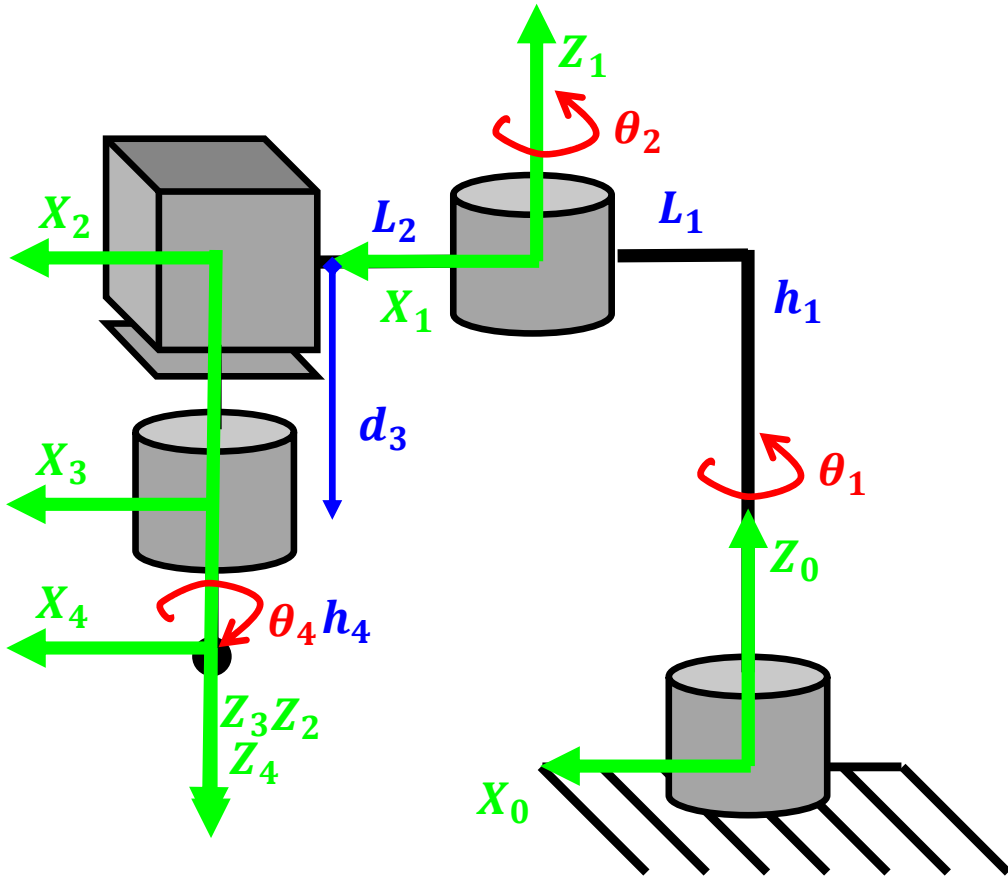
$$J = \begin{bmatrix} -L_1 s_1 - L_2 s_{12} & -L_1 s_1 \\ L_1 c_1 + L_2 c_{12} & L_1 c_1 \end{bmatrix}$$



# Example – SCARA Robot

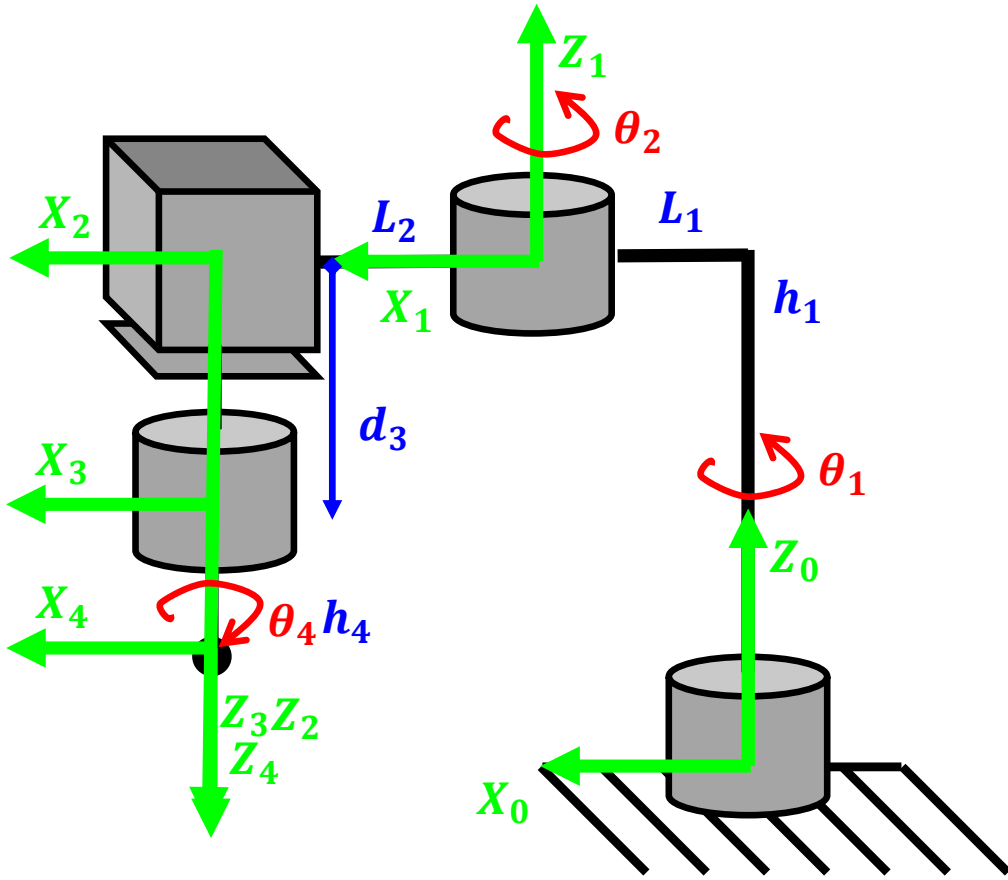
- Given robot as shown, find Jacobian



# Example – SCARA Robot

1) Locate all joint origins $o_i^0$



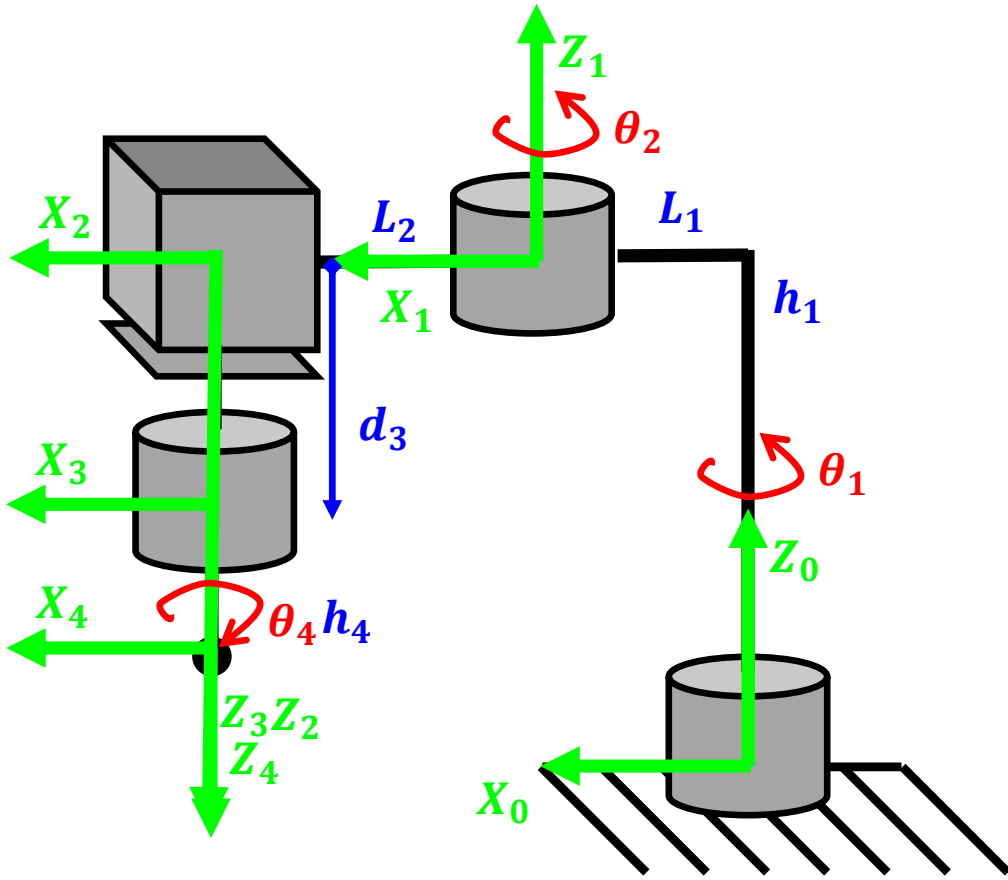
# Example – SCARA Robot

1) Locate all joint origins $o_i^0$

$$o_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad o_2^0 = \begin{bmatrix} L_1c_1 + L_2c_{12} \\ L_1s_1 + L_2s_{12} \\ h_1 \end{bmatrix}$$

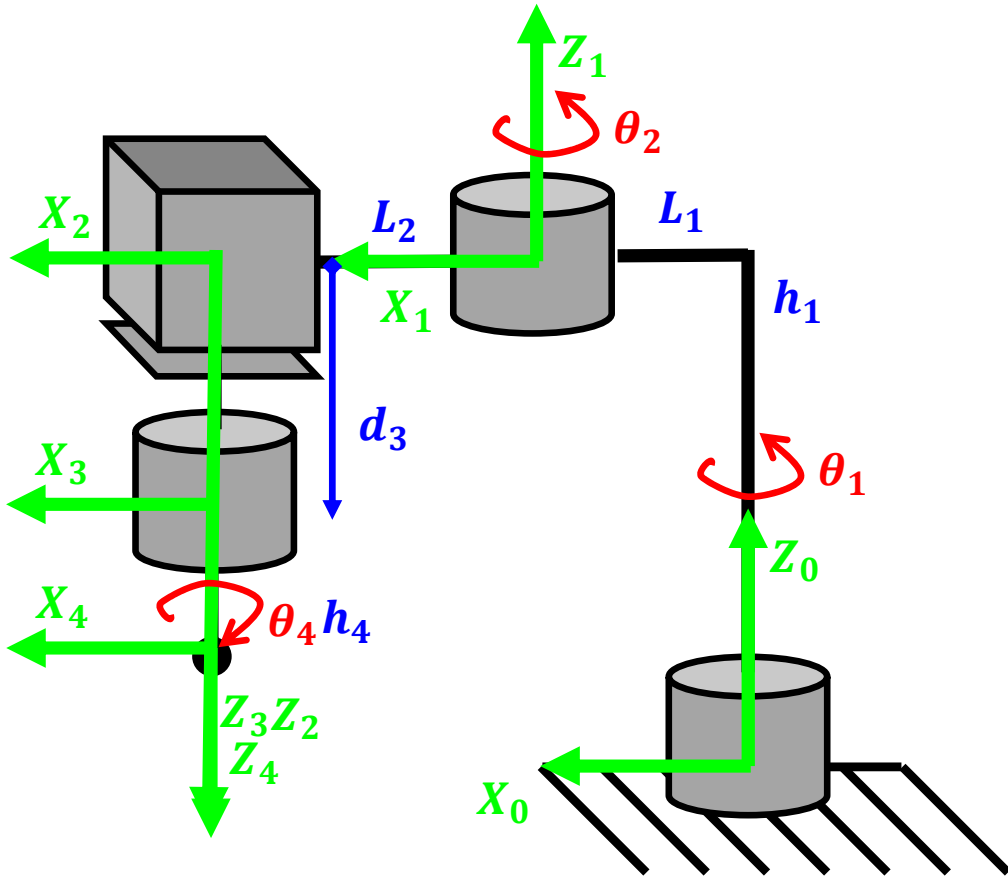
$$o_1^0 = \begin{bmatrix} L_1c_1 \\ L_1s_1 \\ h_1 \end{bmatrix} \qquad o_3^0 = \begin{bmatrix} L_1c_1 + L_2c_{12} \\ L_1s_1 + L_2s_{12} \\ h_1 - d_3 \end{bmatrix}$$

$$o_4^0 = \begin{bmatrix} L_1c_1 + L_2c_{12} \\ L_1s_1 + L_2s_{12} \\ h_1 - d_3 - h_4 \end{bmatrix}$$



# Example – SCARA Robot

2) Write all joint axis vectors $z_i$

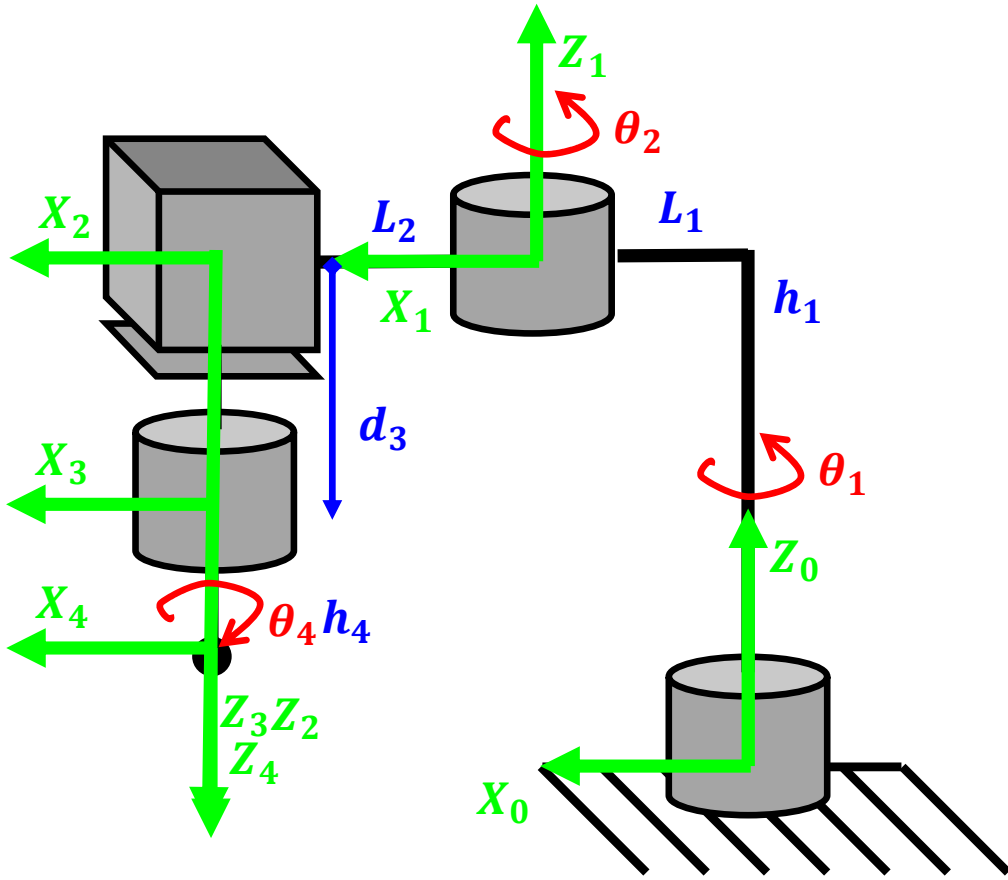


# Example – SCARA Robot

2) Write all joint axis vectors $z_i$

$$z_0^0 = z_1^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

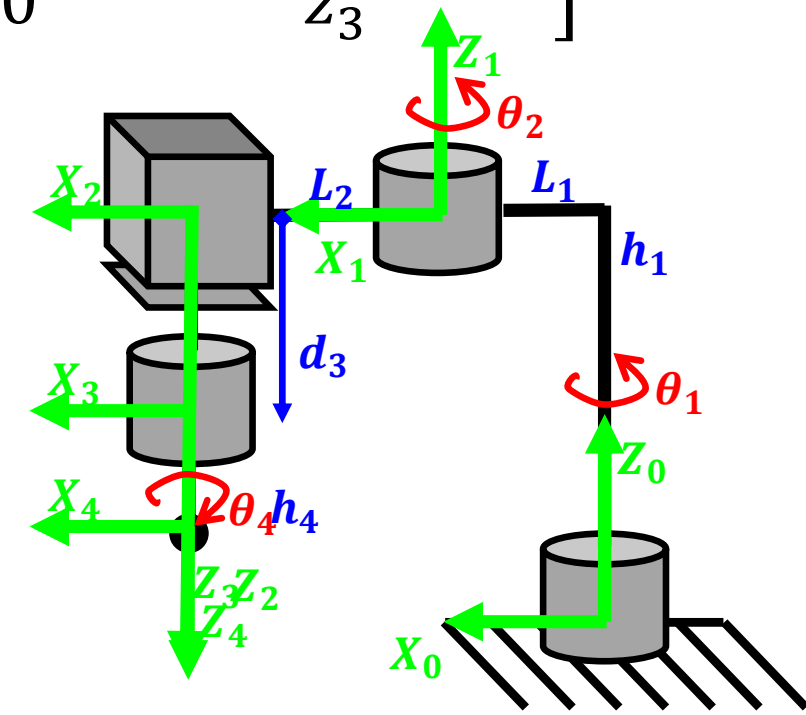
$$z_2^0 = z_3^0 = z_4^0 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$



# Example – SCARA Robot

3) Do math by category (3 R joints, 1 P joint)

$$J = \begin{bmatrix} z_0^0 \times (o_4 - o_0) & z_1^0 \times (o_4 - o_1) & z_2^0 & z_3^0 \times (o_4 - o_3) \\ z_0^0 & z_1^0 & 0 & z_3^0 \end{bmatrix}$$



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$$J = \begin{bmatrix} z_0^0 \times (o_4 - o_0) & z_1^0 \times (o_4 - o_1) & z_2^0 & z_3^0 \times (o_4 - o_3) \\ z_0^0 & z_1^0 & 0 & z_3^0 \end{bmatrix}$$

$$J = \begin{bmatrix} -L_1 s_1 - L_2 s_{12} & -L_2 s_{12} & 0 & 0 \\ L_1 c_1 + L_2 c_{12} & L_2 c_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$