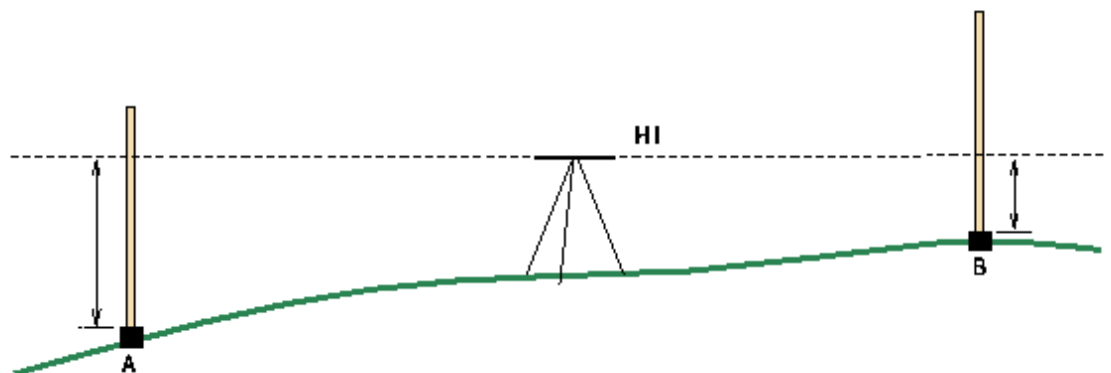


## DIFFERENTIAL LEVELING AND STADIA

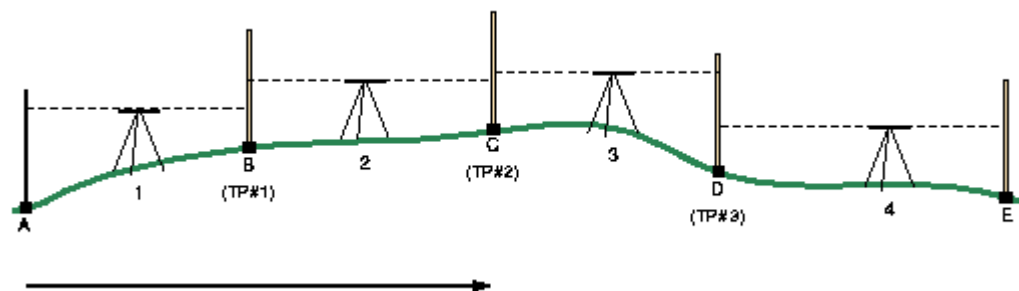
- Differential leveling - Differential leveling is a the process of determining the relative elevations of various points
- Differential leveling notation
  - BS = Basksight
  - FS = Foresight
  - HI = Height of instrument
  - BM = Benchmark (point of known location and elevation)
  - TP = Turning point
- Differential leveling equations
  - $BM+BS = HI$
  - $HI-FS = \text{ELEVATION AT A DESIRED POINT}$
- Uses
  - There are many places where relative elevations are needed. You might be laying out a pipeline, channel, or drainage system and need to make sure that the slope is in the right direction and at a value so that the flow will behave properly.
  - You might be planning a foundation and need to know how much excavation will be required to prepare the sub-grade.
  - The applications in planning and layout are endless.
- Differential leveling basics
  - In differential leveling, the reference "point" that we establish is the height of instrument, **HI**. When the instrument is properly set, we can turn the telescope to any direction and the "line of sight" will be at the same elevation as the scope. So, to apply the differential method, we first measure the vertical distance from Point A up to the line of sight, then measure the vertical distance from Point B up to the line of sight. By comparing these two values, we can determine the difference in elevation between points A & B.



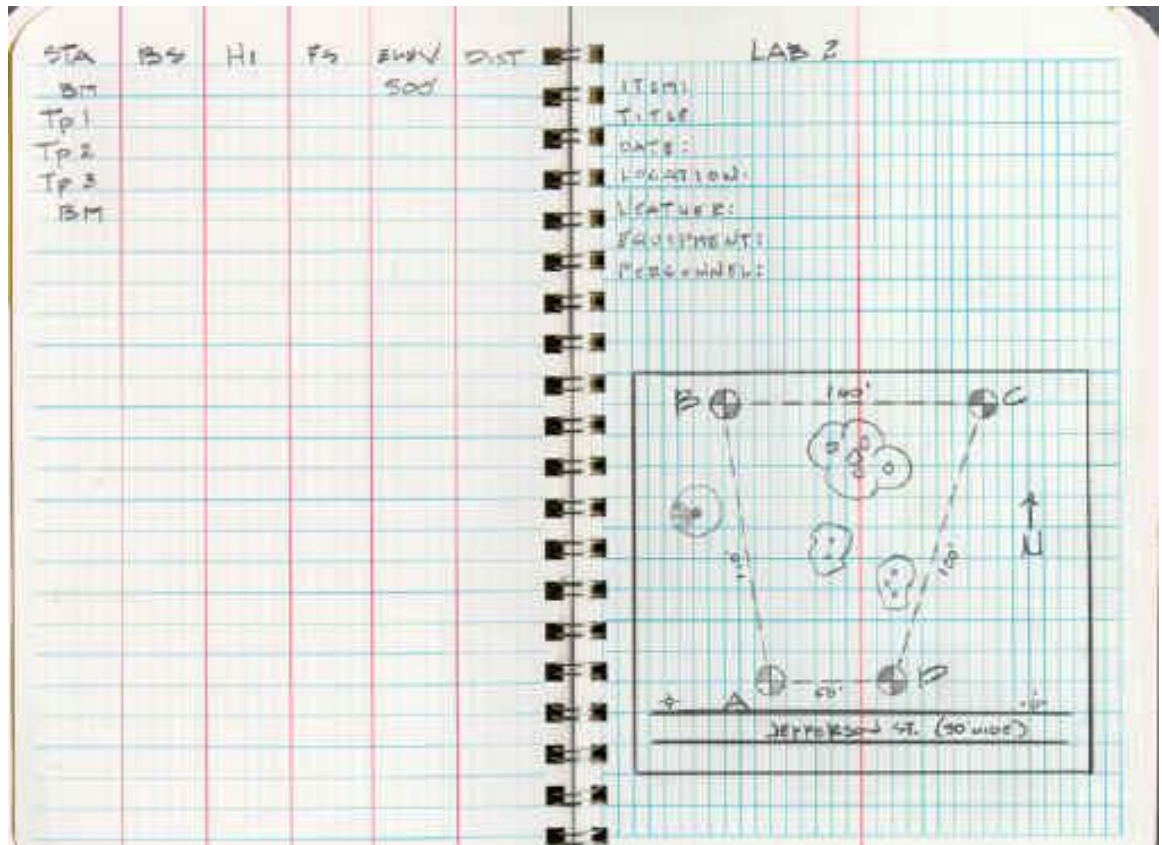
- Example
  - Suppose the measurement at A is 9.26 feet and the measurement at B is 4.18 feet. The difference in elevation from A to B is  $9.26 - 4.18 = 5.08$  feet. Visualize this as "going up 9.26' (from A) to the dotted line (HI),

going sideways along the dotted line without changing elevation to a point directly above B, then going down 4.18' to B." The elevation increment (change) at A was +9.26' and the elevation increment at B was -4.18'. The sum of these increments was 5.08', the net change in elevation from A to B.

- Differential leveling is application of this procedure as many times as needed to obtain the desired result. It could require a single setting of the instrument, or tens or hundreds of settings. The number of settings needed is mainly determined by two factors:
  - 1. The vertical distance between the points. The maximum change in elevation that can be measured at one instrument setting depends on the length of the rod used and the height of the tripod setting.
  - 2. The horizontal distance between the points. With typical instruments and weather conditions, the distance between the instrument and the rod should not exceed about 200 ft (60 m) to maintain accuracy of the readings. On hot days, the atmospheric distortions over paved surfaces (or bare earth) may require reducing the maximum horizontal distance by a third or more.
  - The differential method does not depend on absolute elevations. We do not need to know, for instance, the elevation of Point A relative to mean sea level. However, if the elevation of any point in the survey is known (e.g., a benchmark), then the elevations of all points of the survey can be calculated.
- Procedures
  - In practice, a sequence of instrument settings is often required to determine the required difference(s) in elevation. Suppose the starting point of the survey is Point A. We either know the elevation of A or use an assumed value. (The value "100" is frequently used as an assumed value since it allows considerable room for changes in either direction without running into negative numbers.)
  - The survey will progress forward from A to B, then from B to C, C to D, and D to E. Readings that are taken while looking backward along the survey line are called backsights. Readings that are taken while looking forward along the survey line are called foresights. For example, the reading taken from instrument position 1 to Point A is a backsight and the reading taken from the same instrument position to Point B is a foresight.



- The progression of measurements is shown above. Starting at A, we go up (adding the reading to the elevation at A) to the instrument elevation at the first setting, translate over to B without changing elevation, then go down (subtracting the reading from the instrument elevation) to B. We move the instrument to the second position, go up at B to the new instrument elevation, translate over to C, and to down to C. Move the instrument to the third position, go up at C and down at D. Move to the last instrument position, go up at D and down at E. In this example, Points B, C, and D are merely temporary intermediate points that are used because the distance from A to E is too great to permit a single instrument setting. These intermediate points are Turning Points and will be typically identified as a sequence of TP#1, TP#2, etc., to distinguish them from the "points of interest", namely, A and E.
- Field Notes
  - Column headings on the left page of the field book are:
    - STATION
    - BS
    - HI
    - FS
    - ELEV
  - The name of the point where the reading is taken goes in the STATION column and identifies the values (on that line) which are associated with that point. A backsight taken on that point goes in the BS column. A foresight taken on that point goes in the FS column. The horizontal distance from the previous point to the current point goes in the DISTANCE column. (Actually, it should be the horizontal distance from the previous point to the instrument plus the distance from the instrument to the current point.) For most differential leveling, the distances do not need to be very accurate, so a measurement by pacing will do.
  - It is very important that the readings be entered immediately in their proper places. This provides a clear record of what was done in the field procedure. These procedures have been developed over many years of surveying practice and are universally understood.
  - The HI on any line is the sum of the ELEV and the BS on that line. For the first line, the HI is  $100.00 + 10.21 = 110.21$ . The ELEV on any line (if calculated, not already known or assumed) is the FS on that line subtracted from the HI on the prior line. For the second line (TP #1), the ELEV is  $110.21 - 3.42 = 106.79$ . Refer back to the diagram and the reason for adding or subtracting should become clear.



- Checking the calculations
  - OPEN LOOP FORMULA
    - $\text{Sum(BS)} - \text{Sum(FS)} = \text{Final Elevation} - \text{Initial Elevation}$  or
    - $\text{Initial Elevation} + \text{Sum(BS)} - \text{Sum(FS)} = \text{Final Elevation}$
    - If these do not agree, there is an error in the computations
  - CLOSED LOOP FORMULA
    - $\text{Sum(BS)} = \text{Sum(FS)}$
  - Ideally, the survey values would be perfect. In reality, they are not. Even experienced professionals with top-of-the-line equipment will find some error in their measurements. It simply cannot be avoided. However, it must be controlled
  - Standards have been established for the amount of error considered permissible for different types of projects. You must consider the type of work you are doing and determine the appropriate order of accuracy.
- How much error is allowed?
  - The Federal Geodetic Control Committee specifies the following constants for the formula:
    - Permissible error of closure or allowable misclosure,  $C = [ m * \text{sqrt}(K) ]$ 
      - C is in mm
      - where: K is the distance traversed in kilometers

- “m” is a constant, in mm, obtained from the table below; however, in some cases, we may solve for “m” and determine what order work we have obtained.
  - First-order Class I      4 mm
  - First-order Class II     5 mm
  - Second-order Class I    6 mm
  - Second-order Class II   8 mm
  - Third-order                12 mm
- Example
  - Your survey has traversed 4250'. What is the permissible error of closure for Third Order work?
    - First, convert 4250' to kilometers (1 km = 3281 ft).
      - $4250'/3281' = 1.30$  km
    - Find the value for m in the table: 12 mm.
    - Using the formula given, multiply 12 mm times the square root of 1.30 to get a permissible error of closure for Third Order work of 13.7 mm. That's just over a half inch allowed in 4/5 mile. And remember, Third Order is relatively sloppy work!
- Example
  - You performed differential leveling from station 102A to station 312K and back. The total length of the level run was 9352.36' and you misclosure when back at station 102A was -0.092. What order work does your level run represent?
    - First we need to convert everything to the proper units
      - 9352.36' to km = 2.85059932800 km, this represents our K value
      - .092' to mm = 28.0416 mm, this represents our C value
    - Now we need to determine our “m” value and see where we fall.
      - Our “m” value is 16.087 which falls below third order work
  - In some cases, the number of setups can be fairly large while the distance traversed is relatively small. This often happens on construction sites (and with each setup, the error can increase). An alternative standard for permissible error of closure is "a constant times the square root of the number of setups". For small construction sites, some use the formula:
    - $[2.8\text{mm} * \text{sqrt}(n)]$  where n is the number of setups.
      - Note: Most companies will inform you of what your closure must be; however, if not, ASK!!!!!!

- Elevation adjustment
  - After the allowable error has been computed and the loop misclosure has been determined (final elevation-actual elevation), an elevation adjustment may be made. The adjustment is a cumulative process, beginning with the first computed elevation (do not adjust the initial benchmark). To determine what adjustment is to be applied, divide the loop misclosure by “n” (number of stations, HI’s, or computed elevations)
  - We now have an adjustment per HI, this is the cumulative process. Notice in the example below that our original HI’s do not change. Rather than change the HI’s, we simply apply the correction to the elevations in a cumulative fashion (see example below)
  - Example:

STATION	BS (+)	HI	FS (-)	ELE	ADJ. ELE.
BM MIL	1.33			2053.18	$2053.18 - 0 = 2053.18$
		2054.51			
TP 1	0.22		8.37	2046.14	$2046.14 - (0.004) = 2046.14$
		2046.36			
TP 2	0.96		7.91	2038.45	$2038.45 - 2(0.004) = 2038.44$
		2039.41			
TP 3	0.46		11.72	2027.69	$2027.69 - 3(0.004) = 2027.68$
		2028.15			
BM OAK	11.95		8.71	2019.44	$2019.44 - 4(0.004) = 2019.42$
		2031.39			
TP 4	12.55		2.61	2028.78	$2028.78 - 5(0.004) = 2028.76$
		2041.33			
TP 5	12.77		0.68	2040.65	$2040.65 - 6(0.004) = 2040.63$
		2053.42			
BM MIL	xx		0.21	2053.21	$2053.21 - 7(0.004) = 2053.18$
	Sum = +40.24		Sum = -40.21		

**PAGE CHECK:**

$$2053.18 + 40.24 = 2093.42$$

$2093.42 - 40.21 = 2053.21$  (THIS CHECKS WITH OUR COMPUTED FINAL ELEVATION)

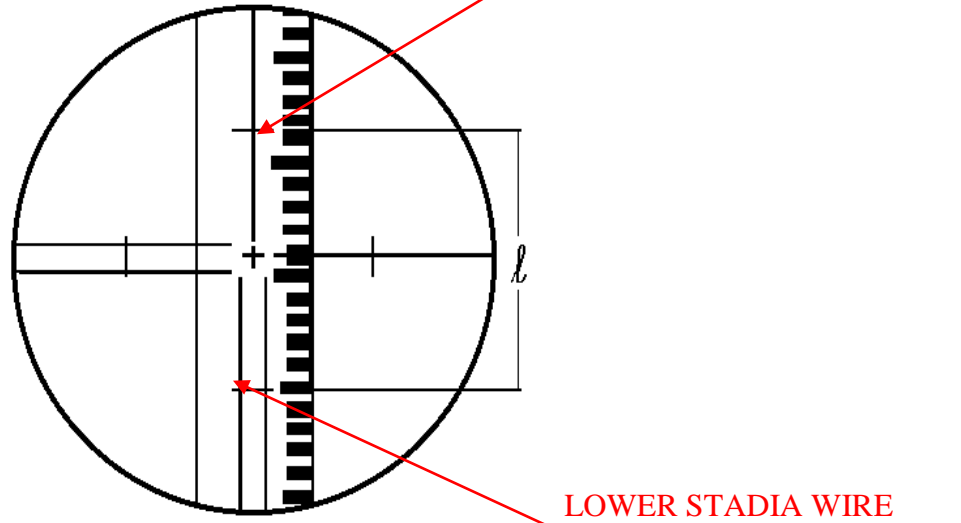
**LOOP MISCLOSURE =  $2053.21 - 2053.18 = 0.03$  (we are high)**

**ADJUSTMENT =  $0.03/7$  (number of stations) =  $0.004/HI$**

**\*\*NOTE\*\* SINCE WE ARE HIGH WE WILL SUBTRACT THE ADJUSTMENT**



- **STADIA** - method of measuring distances rapidly with a telescope (usually on an engineer's transit or an alidade) and a graduated rod.
  - When the telescope is focused on the rod, the distance “s” intercepted on the vertically-held rod between two *stadia hairs* seen in the eyepiece gives the distance D as  $D = (k)*(s)$ , where k, the *stadia constant* is often made to be 100.
  - Example:



- Assume that the upper stadia hair reads 6.36 and the lower stadia hair reads 6.24, this yields a stadia intercept “s”, or “l” in this case, of 0.12 (6.36-6.24). If the stadia constant is 100 the horizontal distance from the instrument to the rod is  $100*s = 12.00'$
- These distances can then be added to give the total perimeter leveled and that value can be used in the Federal Geodetic Control Committee allowable error formula to determine if the work has met a desired tolerance.



**STADIA EXAMPLE: UTILIZING THE EQUATION “C \* sqrt (K)” ?? DID THE CREW MEET 2<sup>ND</sup> ORDER CLASS II WORK**

STATION	BS (+)	HI	FS (-)	ELE	ADJ. ELE.
BM MIL	1.36	UPPER WIRE   MIDDLE WIRE   LOWER WIRE		2053.18	2053.18-0 = 2053.18
	1.33				
	1.30				
	Stadia = 6.0				
		2054.51			
TP 1	0.27		8.45	2046.14	2046.14- (0.004) = 2046.14
	0.22		8.37		
	0.17		8.29		
	Stadia =10.0		Stadia = 16.0		
		2046.36			
TP 2	1.06		8.01	2038.45	2038.45 - 2(0.004) = 2038.44
	0.96		7.91		
	0.86		7.81		
	Stadia = 20.0		Stadia = 20.0		
		2039.41			
TP 3	0.53		11.77	2027.69	2027.69 – 3(0.004) = 2027.68
	0.46		11.72		
	0.39		11.67		
	Stadia = 14.0		Stadia = 10.0		
		2028.15			
BM OAK	12.00		8.75	2019.44	2019.44 – 4(0.004) = 2019.42
	11.95		8.71		
	11.90		8.67		
	Stadia = 10.0		Stadia = 8.0		
		2031.39			
TP 4	12.61		2.71	2028.78	2028.78 – 5(0.004) = 2028.76
	12.55		2.61		
	12.49		2.51		
	Stadia = 12.0		Stadia = 20.0		
		2041.33			
TP 5	12.87		0.71	2040.65	2040.65 – 6(0.004) = 2040.63
	12.77		0.68		
	12.67		0.65		

	Stadia = 20.0		Stadia = 6.0		
		2053.42			
BM MIL	xx		0.26 0.21 0.16  Stadia = 10.0	2053.21	2053.21 – 7(0.004) = 2053.18
	Sum = +40.24		Sum = -40.21		

LOOP MISCLOSURE = 2053.21-2053.18 = 0.03 (we are high)

NOW WE SUM UP THE STADIA OF THE BS AND FS = 182'. SINCE WE NEED THE DISTANCE IN KM CONVERT. 182' = 0.0554736 KM. WE NOW INPUT THE INFORMATION INTO OUR EQUATION: Allowable error = 8mm\*sqrt(0.0554736) = 1.88mm is our allowable error. We had 0.03' to mm = 9.14mm, we exceeded our tolerance.